Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2012

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3A/3B

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

Number of additional answer booklets used (if applicable): ☐

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction tape/fluid, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 12-097

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of total exam
Section One: Calculator-free	8	8	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	11	11	100	100	$66\frac{2}{3}$
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

   Section Two: Write answers in this Question/Answer Booklet. Answer all questions.

   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(5 marks)**

(a) Write down the definition of the derivative of the function $f(x)$. (2 marks)

(b) Use this definition to determine the derivative of $f(x) = x^3$ from first principles. (3 marks)

Question 10 **(12 marks)**

Relative to the origin O, the point A has **Cartesian** coordinates $(\sqrt{3}, 0)$ while the point B has **polar** coordinates $\left(2, \frac{\pi}{6}\right)$.

(a) Write the coordinates of B in Cartesian coordinate form (x_B, y_B). (2 marks)

(b) The point C has Cartesian coordinates (x_C, y_C) such that $x_C = x_B$, $y_C > y_B$ and the angle $\angle COB = 15°$. Determine the coordinates of C in polar form. (4 marks)

See next page

(c) Calculate the areas of the triangles AOB and AOC, leaving your answers in exact form.

(2 marks)

(d) By considering the area of triangle BOC, or otherwise, prove that $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

(4 marks)

See next page

Question 11 **(6 marks)**

The approximate apparent magnitudes of two heavenly bodies are listed in the table below:

Heavenly body	Apparent magnitude m
Sirius	–1.5
Antares	1

The ratio of brightness (or intensity) $\frac{I_A}{I_B}$ of two objects A and B, of apparent magnitudes m_A and m_B respectively, satisfies the equation

$$\log_e\left(\frac{I_A}{I_B}\right) = m_B - m_A \quad .$$

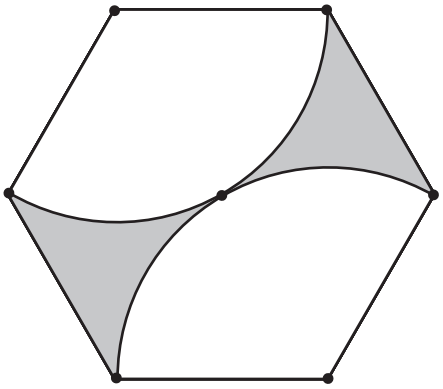
(a) Determine the ratio of brightness of Sirius to Antares, stating your answer to the nearest integer. (3 marks)

(b) If the ratio $\frac{I_{Jupiter}}{I_{Sirius}}$ is $\sqrt{e}$, determine the apparent magnitude of Jupiter. (3 marks)

See next page

Question 12 **(7 marks)**

Consider a regular hexagon with sides of length 6 cm and with inscribed arcs of radii 6 cm as shown.



(a) Determine the exact perimeter of the shaded region. (3 marks)

(b) Determine the shaded area, giving your answer correct to **three (3)** decimal places. (4 marks)

See next page

Question 13 **(8 marks)**

The approximate latitudes and longitudes of three cities are listed below.

Perth, Australia	32S, 116E
Beijing, Peoples Republic of China	40N, 116E
Thessaloniki, Greece	40N, 23E

(a) Assuming the radius of the earth is 6350 km, determine, correct to the nearest kilometre,

(i) the distance between Perth and Beijing, and (3 marks)

(ii) the east-west distance between Thessaloniki and Beijing. (3 marks)

(b) A plane flies from Thessaloniki due east to Beijing and then, after refuelling, flies on to Perth. Assuming that on each part of the journey the plane flies at an average speed of 700 km/h, determine the difference in travel times between the two parts. (2 marks)

See next page

Question 14 **(7 marks)**

The function $f(x)$ is given by the definition

$$f(x) = \begin{cases} x^2 + ax + 1 & \text{if } x \leq 2 \\ 2x^2 + \dfrac{b}{x} & \text{if } x > 2. \end{cases}$$

Determine the constants a and b if the function is continuous and differentiable everywhere.

See next page

Question 15 **(18 marks)**

Consider the two lines L_1 and L_2 defined parametrically by

$$L_1: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{and} \quad L_2: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 4 \end{pmatrix} .$$

(a) Write the equation for L_1 in the form $ax + by + c = 0$. (4 marks)

(b) Determine the position vector of the point of intersection P of the two lines. (4 marks)

(c) Define L_3 to be the line that passes through both the origin (0,0) and the point $Q(1, 2)$.

Determine the equation of the line through Q that is perpendicular to L_3 in the form $ax + by + c = 0$. (5 marks)

(d) Show that the line L_1 is a tangent to the circle $\left|\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix}\right| = \sqrt{5}$. (5 marks)

Question 16 **(9 marks)**

Atmospheric pressure decreases exponentially by 12% for every 1000 m risen above sea level. The pressure P kPa at a height h metres above sea level is given by $P(h) = P_0 e^{kh}$ for some constants k and P_0.

(a) Given that the pressure at sea level is 101.325 kPa, determine (4 marks)

(i) P_0, and

(ii) k (give your answer correct to **three [3]** significant figures).

(b) To the nearest metre, at what height is the pressure half that at sea level? (2 marks)

(c) Élite athletes typically train at high altitudes to improve performance. One training regime is to live at a height of 1800 m and train at a lower altitude of 1000 m. To **two (2)** decimal places, what is the ratio of the pressure at the higher altitude to that at the lower? (3 marks)

See next page

Question 17 **(7 marks)**

(a) Prove $\dfrac{1 - \tan^2 x}{1 + \tan^2 x} = \cos 2x.$ (3 marks)

(b) Hence, or otherwise, show that if $\cos 2\alpha = \tan^2\beta$ then $\cos 2\beta = \tan^2\alpha$. (4 marks)

See next page

Question 18 **(10 marks)**

At 10:15 am a distress call is received at a base station from a ship located at a point with position vector $\mathbf{r} = 40\mathbf{i} + 60\mathbf{j}$ km relative to the base B. The ship drifts with a constant velocity $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$ km/h.

When the call is received the nearest available rescue vessel is situated at $\mathbf{r} = 10\mathbf{i} + 20\mathbf{j}$ km relative to B. The rescue vessel can travel at $\sqrt{89}$ km/h.

(a) On what bearing should the rescue vessel travel in order to meet the ship that made the distress call? (7 marks)

(b) Where and when do the two ships meet? (3 marks)

See next page

Question 19 **(11 marks)**

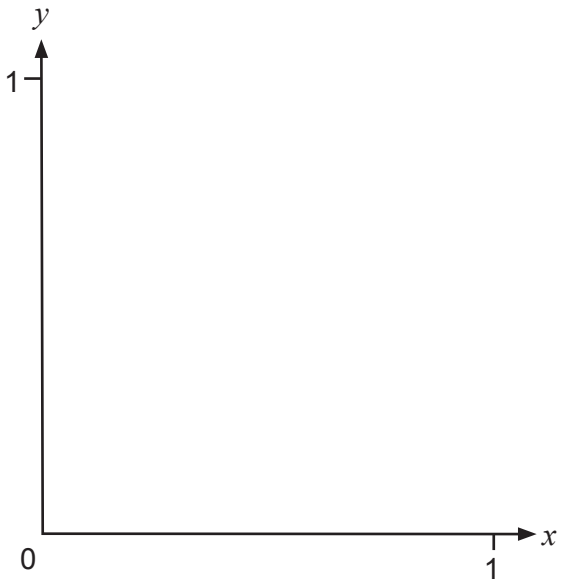
A family of functions $f_n(x)$ with $n = 1, 2, 3...$ is defined by

$$f_n(x) = \frac{1}{n}\,\text{int}[nx] \quad \text{for } x \in [0,1]$$

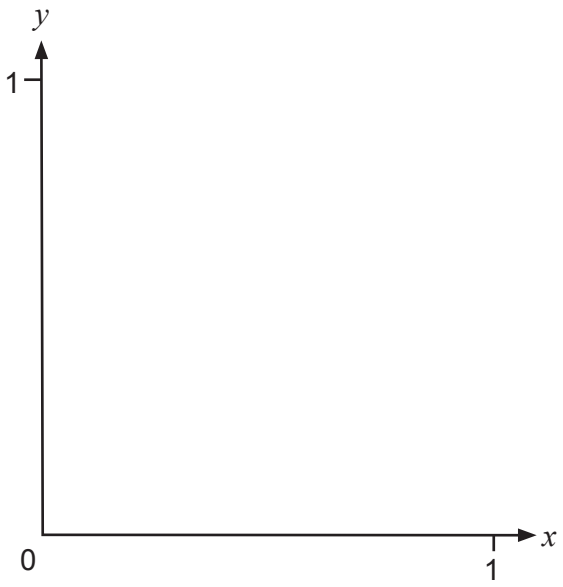
and where int denotes the usual greatest integer function.

(a) On the axes below, sketch the three functions (i) $f_2(x)$, (ii) $f_3(x)$ and (iii) $f_4(x)$. (4 marks)

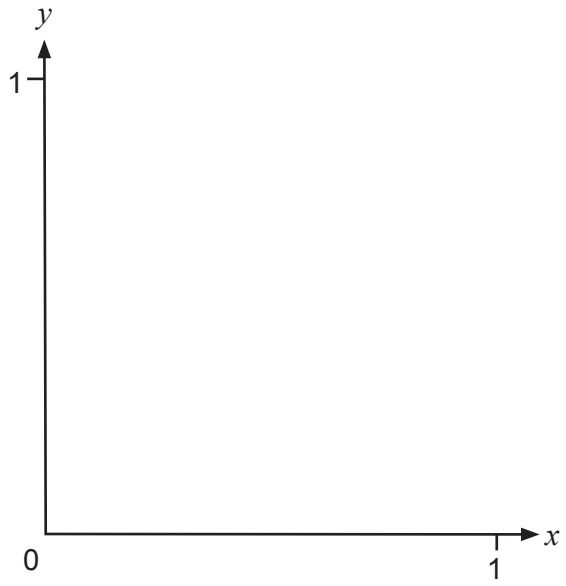
(i)



(ii)



(iii)



(b) Denote by A_n the area bounded by the x-axis, the line $x = 1$ and the function $f_n(x)$. Calculate the values of A_2, A_3 and A_4. (3 marks)

(c) As $n \to \infty$ so the value of A_n approaches a limiting value L. What is the value of L? Give a careful justification of your reasoning. Describe the connection between this result and the integral of a certain function, which should be specified. (4 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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